A Novel Possibility-based Robust Optimal Design Algorithm in Preliminary Design Stage of Electromagnetic Devices

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In the early design stage of an electromagnetic device, sufficient information on uncertainties in design variables is not available. Therefore, a reliable optimal design through conventional reliability analysis by probabilistic method cannot be achieved. This paper, only with insufficient uncertainty data, proposes a new possibility-based optimal design algorithm to get a robust and reliable optimal design of electromagnetic devices. The suggested algorithm adopts a possibility analysis utilizing fuzzy set theory. The possibility analysis employs a surrogate model constructed by design sensitivity analysis to mitigate expensive performance analysis. Finally, the developed optimal design algorithm is validated through applications to several examples.

***Index Terms*—Design optimization, probability, possibility theory, reliability.**

I. Introduction

Very Recently, in the electrical engineering, the reliability-based optimal design (RBOD) algorithms have been proposed to minimize the failure event of a constraint and performance against uncertainties [1]. The RBOD algorithms assume every uncertain design variable follows a certain probability distribution. In early design stage of an electromagnetic device, however, the uncertainty data are inevitably insufficient, and thus a suitable probability density function cannot be obtained [2]. In addition, the improper modeling of uncertainties may cause overestimation of reliability [3]. It is very essential, therefore, to develop a new algorithm guaranteeing a reliable optimal design even with insufficient uncertainty data. In structural engineering, the possibility analysis has been applied to find reliable optimal design against insufficient uncertainty data [2], [4]. In electromagnetic field, however, this is a relatively new topic and has not been researched yet.

To deal with insufficient uncertainty data, this paper proposes a novel possibility-based robust optimization algorithm suitable for electromagnetic problems. In the proposed algorithm, the possibility analysis based on fuzzy set theory is developed to perform a reliability analysis and combined with particle swarm optimization to search for reliable design. During the possibility analysis, to reduce the computational cost resulted from expensive performance analysis, a surrogate model assisted by sensitivity analysis is constructed and incorporated. Finally, the searching ability of the proposed robust algorithm is investigated and compared with RBOD.

II. Possibility-Based Robust Optimal Design

In the RBOD, the reliability of a design is evaluated as the probability of satisfying performance constraint [1]. Its mathematical formulation is written as:

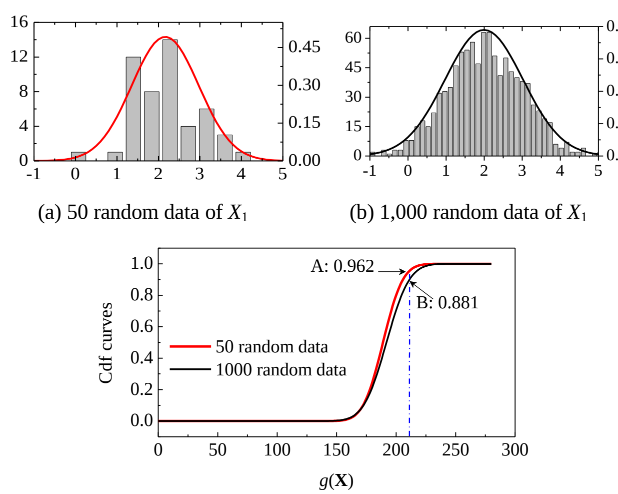
$$\begin{aligned} &\text{Minimize} \quad f(\mathbf{d},\mathbf{x},\mathbf{p}) \\ &\text{subject to} \quad P\left\{g_i\left(\mathbf{d},\mathbf{X},\mathbf{P}\right)\le 0\right\}\ge R_{i,t}, \quad i=1,\cdots,nc \end{aligned} \tag{1}$$

where $f(\cdot)$ and $g(\cdot)$ are objective and constraint functions, respectively; nc is the number of constraints; $R_{i,t}$ is the target reliability of i-th constraint; $\mathbf{d}$ and $\mathbf{x}$ are vectors of nominal values for deterministic and uncertain design variables which are to be optimized, respectively, while $\mathbf{p}$ is uncertain parameters with fixed nominal values and not to be optimized; $\mathbf{X}$ and $\mathbf{P}$ are vectors of uncertain regions around $\mathbf{x}$ and $\mathbf{p}$, respectively.

Due to lack of knowledge or information on data, it is difficult to calculate probability function $P(.)$ in (1) [4]. In this case, the RBOD may fail to lead to a reliable design. An example in Fig. 1, is used to show the shortcoming of probabilistic method in RBOD. From Fig. 1(c), it is obvious that the probability of satisfying constraint $g(\mathbf{X}) \le 212$ calculated from insufficient data is 96.2% while that calculated from sufficient data is only 88.1%. It can be said the probabilistic method is out of work under insufficient information.

A. Possibility Analysis based on Fuzzy Set Theory

In the fuzzy theory, for insufficient uncertainty data, it is much easier to decide lower and upper bounds. Then, the uncertainty set of random variable $U(\mathbf{x})$ is decided by the α-cut, which is the range of values of that variable that have possibility equal or greater than α defined as:



(a) 50 random data of X_1 (b) 1,000 random data of X_1

(c) Fitted CDF curves of performance function with statistic information
50 data: X_1~N (2.164, 0.810), X_2~N (3.785, 0.879)
1000 data: X_1~N (1.996, 1.000), X_2~N (4.023, 1.002)

Fig. 1. Analytic example of reliability analysis with insufficient data, where constraint is $g(\mathbf{X})=X_1+12X_2+130$, and standard deviation of $\mathbf{X}$ is $\boldsymbol{\sigma}$=1.0.

$$U(\mathbf{x}_\alpha) = \left\{\mathbf{x} \middle| \mathbf{x}_{L,\alpha} \le \mathbf{x} \le \mathbf{x}_{U,\alpha}, \alpha \in [0,1]\right\} \quad (2)$$

where $\mathbf{x}_{L,\alpha}$ and $\mathbf{x}_{U,\alpha}$ are the lower and upper bounds respectively. For an electromagnetic system, the random input variables will generate a stochastic response as shown in Fig. 2. Then, the possibility of a failure event is equal to the maximum value of possibilities of combinations of input values that correspond to failure. The constraint $g(\mathbf{d},\mathbf{X},\mathbf{P})\le 0$ in (1) defines a feasible region, therefore, for a specific problem shown in Fig. 2, the possibility of constraint violation is:

$$\Pi\left(g \ge 0\right) = \alpha_1 . \quad (3)$$

where $\Pi(\cdot)$ is the possibility distribution function [5]. If the α-cut level is very small (α<<1), (3) can be relaxed as:

$$\Pi\left(g \ge 0\right) \le \alpha \text{ or } \Pi\left(g \le 0\right) \ge 1-\alpha . \quad (4)$$

Furthermore, for simplicity, the possibility constraint in (4) can be transferred into a simple nominal constraint as follows:

$$g^{\alpha}_{\max} \le 0 . \quad (5)$$

where $g^{\alpha}_{\max}$ is global maximum constraint value at the α-cut [6]. Finding a design satisfying given possibility level, therefore, is to find $g^{\alpha}_{\max}$, which satisfies nominal constraint.

B. Proposed Possibility-Based Optimal Design (PBOD)

A conservative optimum design is preferred when accurate statistical information is not available, so the possibility analysis may be a desirable merit. The PBOD is formulated as:

$$\begin{aligned} &\text{Minimize} \quad f(\mathbf{d},\mathbf{x},\mathbf{p}) \\ &\text{subject to} \quad \Pi\left\{g_i\left(\mathbf{d},\mathbf{X},\mathbf{P}\right) \le 0\right\} \ge 1-\alpha, \quad i = 1,\cdots,nc \end{aligned} \quad (6)$$

where possibility constraint means the possibility of satisfying constraint should be bigger than the predefined (1-α)-level. In optimization process, the possibility constraint is replaced by its equivalent form (5). If (5) is satisfied, in the particle swarm optimizer, the corresponding design will be accepted for next iteration to search for the global reliable optimal solution.

As shown in (5), the PBOD problem belongs to a double-loop optimization. To mitigate expensive computational cost, for a specified design (**d**, **x**), the performance of any fuzzy combination ($\xi_{\mathbf{x}}, \xi_{\mathbf{p}}$) in the α-cut is approximated as:

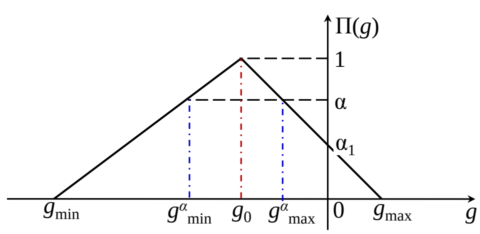


Fig. 2. Random response g and its membership function [6].

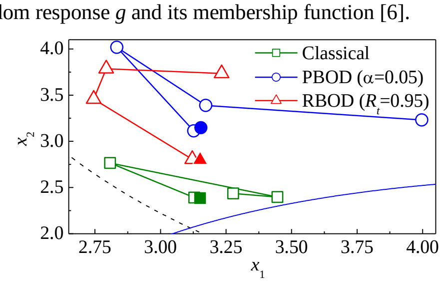


Fig. 3. Comparison of different designs (filled markers are optimal ones).

$$g(\xi_{\mathbf{x}},\xi_{\mathbf{p}}) \cong g(\mathbf{d},\mathbf{x},\mathbf{p}) + \nabla g_{\mathbf{x}} \cdot (\xi_{\mathbf{x}} - \mathbf{x}) + \nabla g_{\mathbf{p}} \cdot (\xi_{\mathbf{p}} - \mathbf{p}) \quad (7)$$

where the gradient vector is obtained by the adjoint variable method. Once constraint is approximated by (7), the particle swarm optimizer is employed to (7) and (6) searching for both the worst constraint in the inner loop and reliable optimal design in the outer loop, respectively.

III. Performance Investigation of PBOD

A. Analytic Example

The analytic problem is defined as:

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) = \sin(3x_1^2) + \sin(3x_2^2) + x_1 + x_2 \\ \text{subject to} \quad & g_1(\mathbf{X}) = 1 - X_1^2 X_2 / 20 \le 0 \\ & g_2(\mathbf{X}) = 1 - s^2/30 - t^2/120 \le 0 \\ & g_3(\mathbf{X}) = 1 - 80 / \left(X_1^2 + 8X_2 + 5\right) \le 0 \end{aligned} \quad (8)$$

where the design space is $0 \le x_1, x_2 \le 10$, $s = X_1+X_2-5$ and $t = X_1-X_2-12$. The deterministic optimum is $\mathbf{x} = (3.1513, 2.3951)$.

In RBOD, the uncertain design variables are assumed following $\mathbf{X}\sim N(\boldsymbol{\mu}=\mathbf{x}, \boldsymbol{\sigma}=0.2)$. In PBOD, the fuzzy set of zero α-cut is defined as [$\mathbf{x}$-2.063$\boldsymbol{\sigma}$, $\mathbf{x}$+2.063$\boldsymbol{\sigma}$]. In Table I, as α-level increases, the PBOD is much closer to classical optimum. By comparing searching directions in Fig. 3, the possibility optimum is more conservative than the probability optimum.

B. Electromagnetic Problem –TEAM 22

In the TEAM problem 22 [1], three geometric variables of outer coil $\mathbf{x}=[R_2, H_2/2, D_2]^T$ are treated as Gaussian random design variables caused by manufacturing tolerance. The fuzzy set is set as 3σ level and $\boldsymbol{\sigma}=[0.0153, 0.01, 0.01]^T$ m.

As shown in Table II, compared with classical optimum, both RBOD and PBOD can find reliable designs further from constraint boundary. Furthermore, if the α-cut level of 0.05 is equivalent to a target failure probability of 0.05 ($1-R_t$), the PBOD is also more conservative than the RBOD. As the α-cut level decreases, the optimal design shows more conservatism.

IV. References

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TABLE I Optimal Results Comparison of RBOD and PBOD

Methods	Parameter	x_1	x_2	$f(\mathbf{x})$	$g_1(\mathbf{x})$	$g_2(\mathbf{x})$	$g_3(\mathbf{x})$
Classical	-	3.1513	2.3951	3.5502	-0.1892	-0.0634	-1.3466
RBOD	R_t=0.95	3.1513	2.7989	3.9534	-0.3897	-0.1606	-1.1434
PBOD	α=0.05	3.1355	3.1523	4.3492	-0.5496	-0.2586	-0.9975
	α=0.01	3.1443	3.4679	4.6305	-0.7143	-0.3522	-0.8766

TABLE II Optimization Results of TEAM 22 by Different Methods

Method	Parameter	R_2	$H_2/2$	D_2	$f(\mathbf{x})$(E-2)	$g_1(\mathbf{x})$	$g_2(\mathbf{x})$
Classical	-	3.0819	0.2439	0.3849	8.7719	-7.896	-1.384
RBOD	R_t=0.95	3.0892	0.2669	0.3500	8.8169	-7.779	-2.076
PBOD	**α=0.05**	**3.1106**	**0.3068**	**0.3004**	**9.0611**	**-7.560**	**-3.600**
	α=0.01	3.1151	0.3155	0.2915	9.1343	-7.519	-3.928